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Transverse ordering in off-critical quenches of a system with continuous symmetry

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Abstract. The dynamics of a system quenched below the critical point with explicitly broken symmetry (off-critical quench) is considered in the framework of an O(N) vector model in the large-N limit with non-conserved order parameter. Considering the behaviour of fluctuations in the transverse directions, we find ordering and scaling of the transverse structure factor in the intermediate time regime between the usual early and late stages.

1. Introduction

In the time evolution of a system quenched below the critical point one usually makes the distinction between early and late stages. Immediately after the quench there is exponential growth of fluctuations, well described by linear theories [1]. Later on, when non-linearities become effective, growth slows down, domains are formed with locally broken symmetry and the process of phase ordering takes place†. In the late stage of this process local equilibrium is reached and, within domains, the order parameter saturates to one of the allowed ground-state values. Late-stage theories [2] are built on the observation that in this time regime the only dynamics left in the system is the motion of domain walls. The equivalent statement is that all time dependence takes place through the average domain size $L(t)$ produced as a consequence of dynamic scaling [3], a phenomenon which is the object of intensive investigation but, as yet, not fully understood.

By contrast, very little is known about the time regime in between the early and late stages. According to the picture outlined above, this intermediate stage should occur when domains have already formed, but the order parameter has not yet reached saturation and fluctuations do play a significant role. Although the identification and study of such a regime, in general, is a difficult problem [4], investigation of off-critical quenches has revealed that non-trivial phenomena with universal features also take place before the asymptotic regime is reached. In particular Janssen *et al* [5] have discovered a new exponent characterizing the behaviour of the magnetization in the intermediate stage of a quench from high temperature to the critical point. Anomalous transverse fluctuations with scaling behaviour in the intermediate stage have also been found in the somewhat different context of the magnetization instability following an isothermal magnetic field inversion [6, 7].

Here we consider off-critical quenches below the critical point in a system with a vectorial, non-conserved order parameter in the large-N limit [8]. In processes of this type symmetry is explicitly broken from the beginning either by preparing the system in an unsymmetrical initial state or by turning on a small external field immediately after the quench. In either case a non-zero expectation value of the order parameter (magnetization

† For simplicity the physical picture is presented here with the terminology of domain formation, which is appropriate only for scalar order parameters.

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for short) develops over the entire volume of the system and in the same direction. In so doing the whole system acts like one single domain in the phase-ordering problem. Clearly the notion of domain wall (more generally topological defect) disappears here and the late stage is trivial but, as we shall see below, there is a non-trivial intermediate stage characterized by the growth and scaling of fluctuations in the transverse direction.

2. Formalism

The time evolution of the system is described by the equation of motion of the Langevin type

$$\frac{\partial \phi(\boldsymbol{x},t)}{\partial t} = \frac{1}{2}\left[\nabla^2\phi + r\phi - \frac{g}{N}(\phi^2)\phi + \boldsymbol{h} + \beta(\boldsymbol{x},t)\right] \tag{2.1}$$

where $\phi = (\phi_1, \ldots, \phi_N)$ is an N-component order parameter, r and g are positive quantities, $\boldsymbol{h}$ is an external field and $\beta(\boldsymbol{x},t)$ is the Gaussian white noise with expectations

$$\langle \eta(\boldsymbol{x},t)\rangle = 0 \qquad \langle \eta_\alpha(\boldsymbol{x},t)\eta_\beta(\boldsymbol{x}',t')\rangle = 2T\delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t') \tag{2.2}$$

where T is the final equilibrium temperature. In the following, symmetry will be broken along the 1-axis either by preparing an initial state with non-vanishing magnetization in that direction or by applying a non-vanishing field of the form $h_\alpha = h\delta_{\alpha 1}$.

In both cases, with the magnetization $m(t)$ defined by

$$\langle \phi_\alpha(\boldsymbol{x},t)\rangle = N^{1/2} m(t)\delta_{\alpha 1} \tag{2.3}$$

the transverse and longitudinal equal-time correlation functions are given by

$$\begin{aligned} &C_\parallel(\boldsymbol{x}-\boldsymbol{x}',t) = \langle \phi_1(\boldsymbol{x},t)\phi_1(\boldsymbol{x}',t)\rangle - Nm^2(t) \\ &C_\perp(\boldsymbol{x}-\boldsymbol{x}',t) = \langle \phi_\alpha(\boldsymbol{x},t)\phi_\alpha(\boldsymbol{x}',t)\rangle \\ &\alpha \neq 1 \end{aligned} \tag{2.4}$$

where averages are taken over the initial state and thermal noise. By Fourier transforming and taking the $N \to \infty$ limit these quantities obey the set of equations [6]

$$\partial m(t)/\partial t = -\tfrac{1}{2}\left[-r + g(m^2(t) + S(t))\right] m(t) + \tfrac{1}{2}h \tag{2.5}$$

$$\partial C_\parallel(\boldsymbol{k},t)/\partial t = -\left[k^2 - r + g(3m^2(t) + S(t))\right] C_\parallel(\boldsymbol{k},t) + T \tag{2.6}$$

$$\partial C_\perp(\boldsymbol{k},t)/\partial t = -\left[k^2 - r + g(m^2(t) + S(t))\right] C_\perp(\boldsymbol{k},t) + T \tag{2.7}$$

which is closed by the self-consistency condition

$$S(t) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} C_\perp(\boldsymbol{k},t). \tag{2.8}$$

Since the longitudinal structure factor $C_\parallel(\boldsymbol{k},t)$ is completely determined by $m(t)$ and $C_\perp(\boldsymbol{k},t)$, from now on we shall concentrate on the pair of coupled equations (2.5) and (2.7) with initial conditions $m(0) = m_0$ and $C_\perp(\boldsymbol{k},0) = \Delta$.

Let us briefly summarize the zero-field equilibrium properties [6] which will be needed in the following. Imposing a momentum cutoff at $k = 1$, there is a critical temperature given by $T_c = 2^{d-1}\pi^{d/2}\Gamma(d/2)(d-2)r/g$, where d is the space dimensionality. Below T_c the spontaneous magnetization is given by

$$m^2(\infty) = \left(\frac{T_c - T}{T_c}\right)\frac{r}{g} \tag{2.9}$$

while the transverse structure factor

$$C(\boldsymbol{k}, \infty) = T/k^2 \tag{2.10}$$

exhibits the gapless $\boldsymbol{k}$-dependence due to the presence of the Nambu–Goldstone modes.

Going back to the dynamical problem, equations (2.5) and (2.7) can be formally integrated yielding

$$m(t) = m_0 e^{-Q(t)/2} + \tfrac{1}{2}h\int_0^t dt' e^{-[Q(t)-Q(t')]/2} \tag{2.11}$$

$$C_\perp(\boldsymbol{k}, t) = \Delta e^{-[k^2 t + Q(t)]} + T\int_0^t dt' e^{-[k^2(t-t')+Q(t)-Q(t')]} \tag{2.12}$$

where

$$Q(t) = \int_0^t dt' \,[-r + g(m^2(t') + S(t'))]. \tag{2.13}$$

In a paper devoted to the determination of the λ exponent [9] from the behaviour of the magnetization, Bray and Kissner [10] have analysed, in detail, an equation of the type (2.11). Our primary concern here is with the behaviour of the transverse structure factor $C_\perp(\boldsymbol{k}, t)$. The central quantity is $Q(t)$. Defining the length $L(t) = t^{1/2}$, from equation (2.12) follows the scaling behaviour

$$C(\boldsymbol{k}, t) \sim L^a(t) f(x) + TL^2(t) g(x) \tag{2.14}$$

with $x = kL(t)$ and

$$f(x) = e^{-x^2} \qquad g(x) = 2x^{d-2}e^{-x^2}\int_0^{x^2} dx'\, x'^{(1-d)} e^{x'^2} \tag{2.15}$$

over time intervals where the power law

$$e^{-Q(t)} \sim L^a(t) \tag{2.16}$$

is obeyed, with some exponent a to be determined. In order to establish whether this will occur at all we must analyse the equation for $Q(t)$ obtained via the definition (2.13) and the self-consistency condition (2.8)

$$\frac{d}{dt}e^{Q(t)} = -re^{Q(t)} + \left[m_0 + \frac{h}{2}\int_0^t dt' e^{Q(t')/2}\right]^2 + \Delta A(t) + T\int_0^t dt'\, A(t-t') e^{Q(t')} \tag{2.17}$$

where

$$A(t) = \int \frac{d^d k}{(2\pi)^d} e^{-k^2 t}. \tag{2.18}$$

3. Off-critical quenches

Let us first consider a quench without external field ($h = 0$) at zero final temperature ($T = 0$). In that case the amplitude of the transverse structure factor is given by

$$C_{\perp}(k = 0, t) = \Delta e^{-Q(t)}. \tag{3.1}$$

Then, for short time ($t \ll r$), equation (2.17) yields the early-stage exponential growth anticipated in the introduction

$$C_{\perp}(k = 0, t) \simeq e^{rt} \tag{3.2}$$

while, for longer times, one has

$$C_{\perp}(k = 0, t) \simeq \frac{m(\infty)}{\left[1 + (\Delta c/m_0^2)t^{-d/2}\right]} \tag{3.3}$$

where c is a positive constant. This result shows that there exists a characteristic time related to the initial condition

$$t^* = \left(\Delta c/m_0^2\right)^{2/d} \tag{3.4}$$

such that if $t^* \gg 1/r$ it is possible to find an intermediate temporal regime ($1/r \ll t \ll t^*$) where the power law (2.16) is obeyed with $a = d$. Beyond that, in the late stage, the amplitude relaxes to the finite value $C_{\perp}(k = 0, \infty) = \Delta m^2(\infty)/m_0^2$. The interesting finding then is the existence of a time interval (intermediate stage) where the scaling form (2.14) holds. This is the counterpart, for the transverse structure factor, of the scaling regime found by Bray and Kissner for the magnetization before saturation.

The duration of this regime can be modulated by varying initial conditions. In particular, it should be noted that t^* diverges as m_0 goes to zero. Thus, from the point of view adopted here, symmetrical critical quenches may be regarded as the limiting cases where the magnetization stays identically zero, there is no distinction of longitudinal and transverse modes and the duration of the scaling regime is unbounded. Since in that case scaling is associated with ordering, we may conclude that the existence of an intermediate scaling regime in off-critical quenches, as described above, is a manifestation of ordering in the transverse directions. Namely, as long as the magnetization is below saturation, transverse fluctuations grow much in the same way as if symmetry had not been broken. It is only near saturation that fluctuations regress and eventually disappear.

In order to follow the time sequence of the three stages above mentioned, equations (2.5) and (2.7) have been solved numerically for $d = 3$ with $r = 10$, $g = 1$ (corresponding to $T_c = 197.4$) and initial conditions $m_0 = 0.001$, $\Delta = 1$ (which give $t^* \sim 10^3$). In figure 1 the behaviour of $C_{\perp}(k = 0, t)$ is plotted showing agreement with the exponential growth law (3.2) at short time. In figure 2 the behaviour of the amplitude $C_{\perp}(k = 0, t)$ is displayed over a much longer time interval in a double logarithmic plot. For comparison (broken curve) the behaviour of the amplitude in the symmetrical case ($m_0 = 0$) is also reported. For $t < t^*$ the plot clearly shows the existence of an intermediate stage, precisely for $1 < t < 10^2$, where the amplitude scales like $L^3(t)$ exactly as in the symmetrical case, revealing growth of order in the transverse directions. For $t \sim t^*$ deviation from scaling

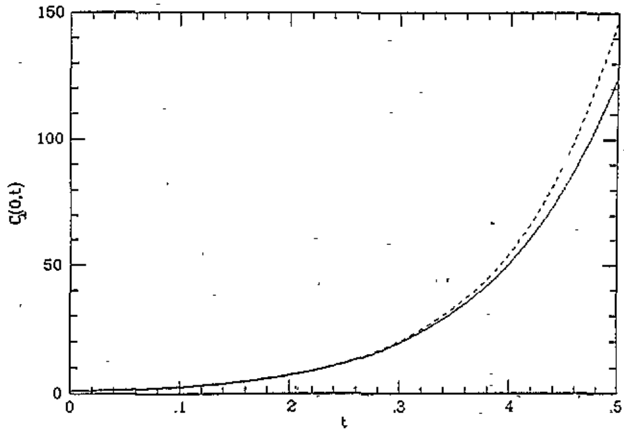

Figure 1. Exponential growth of the transverse amplitude (full curve)in the early stage of a quench with $m_0 = 0.001, \Delta = 1, T = 0, h = 0$. The broken curve is the amplitude in the symmetrical case ($m_0 = 0$) which goes like Δe^{rt}.

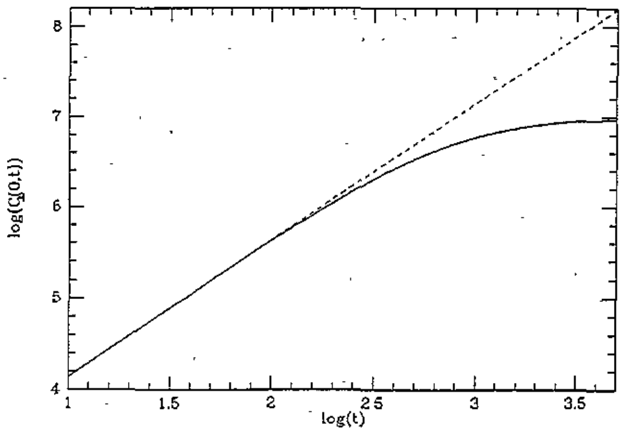

Figure 2. Ordering in the transverse directions in the intermediate stage of a quench with $m_0 = 0.001, \Delta = 1, T = 0, h = 0$. The slope of the broken line is 1.5.

occurs and eventually the late stage is entered where relaxation to the final equilibrium value takes place.

This picture changes in a quench to a finite final temperature. In that case the transverse structure factor must eventually relax to the equilibrium form (2.10), with a scaling behaviour described by the second term in the right-hand side of (2.14). Therefore we expect the same behaviour as in a zero temperature quench as far as the early and intermediate stages are concerned, namely ordering in the transverse directions, followed by a late stage where the amplitude $C_\perp(k = 0, t)$ rather than crossing over to a constant value, scales like $L^2(t)$ signalling the onset of the Nambu–Goldstone modes. This is illustrated in figure 3 for a quench at $T = 3T_c/4$.

Let us now consider the numerical solution of equations (2.5) and (2.7) in the presence of a small external field along the 1-axis. Specifically we consider a quench with $r = 10, g = 1, T = T_c/2, h = 0.001$ and symmetrical initial conditions $m_0 = 0, \Delta = 0.01$. The behaviour of the transverse amplitude is displayed in figure 4. Omitting the early stage, which does not show in the scale of the figure, again the plot shows clearly the

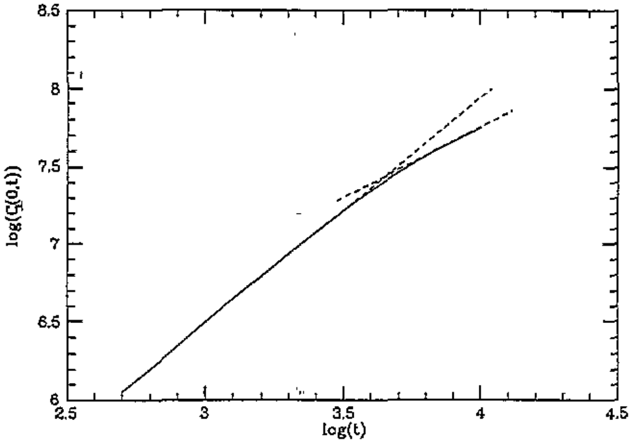

Figure 3. Crossover from ordering in the transverse directions to the growth of the Nambu–Goldstone modes in a quench with $m_0 = 0.001$, $\Delta = 0$, $T = \frac{3}{4}T_c$, $h = 0$. The slopes of broken curves are 1.45 in the intermediate stage and 0.9 in the late stage.

existence of an intermediate stage where the transverse amplitude scales like $L^3(t)$ as in the symmetrical case (broken curve). Eventually the late stage is entered where relaxation to the final equilibrium value $C_\perp(k = 0, \infty) = Tm(\infty)/h$ takes place. In this case there are no Nambu–Goldstone modes, since the transverse correlation length remains finite.

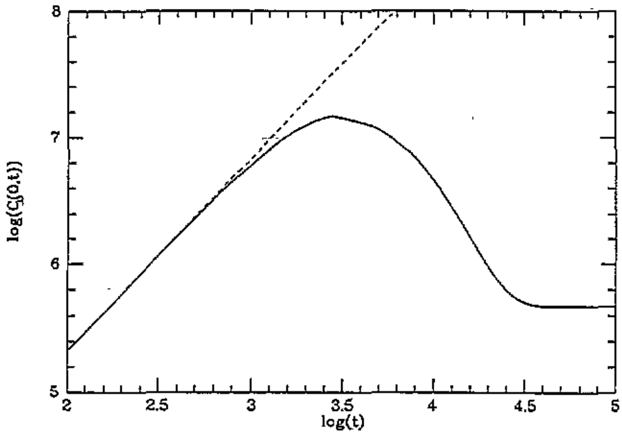

Figure 4. Ordering in the transverse directions in the intermediate stage of a quench with $m_0 = 0$, $\Delta = 0.01$, $T = T_c/2, h = 0.001$. The slope of the broken line is 1.45.

In order to show that the behaviour of the transverse structure factor in the intermediate stage is controlled by the first term in the right-hand side of (2.14), in figure 5 we have rescaled data taken at five different times obtaining an excellent data collapse on a scaling function of the form

$$f(x) = \frac{1}{L^3(t)} C_\perp \left(\frac{x}{L(t)}, t \right) = 5.35\mathrm{e}^{-0.92x^2}. \tag{3.5}$$

Finally, it is clear that the intermediate stage becomes longer as the external field becomes smaller. Estimating the order of magnitude of the duration of the intermediate stage by the time it takes for the amplitude to reach its maximum value (figure 4), namely by $t_{\max}$ defined by $C_\perp(k = 0, t_{\max}) = \max$, we find (figure 6) the power law $t_{\max}^{-1} = 0.2h^{0.9}$.

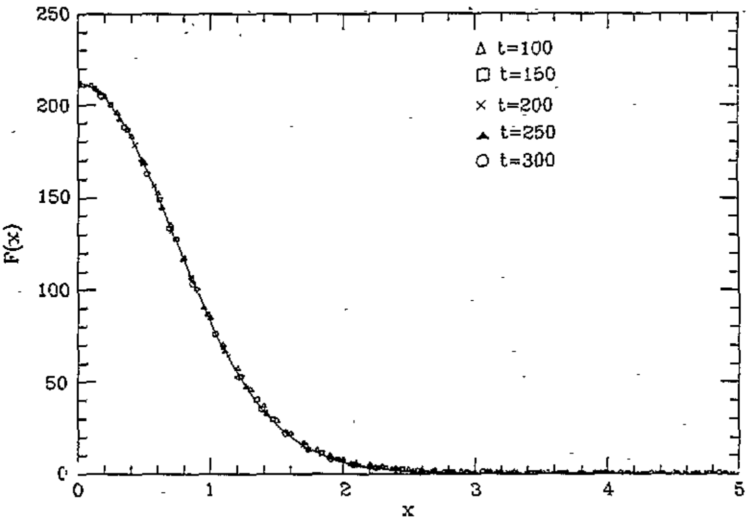

Figure 5. Data collapse for the transverse structure factor in the intermediate stage of a quench with $m_0 = 0$, $\Delta = 0.01$, $T = T_c/2, h = 0.001$. The full curve is the plot of $f(x)$ defined by equation (3.5).

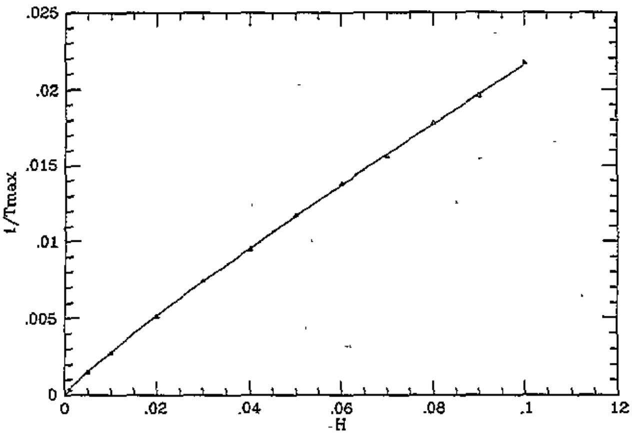

Figure 6. Plot of t_{max}^{-1} against h. The full curve is best fit with $t_{max}^{-1} = 0.2h^{0.9}$.

4. Concluding remarks

In conclusion, by considering off-critical quenches obtained either by preparing the system with a small initial magnetization or by applying a small external field we have shown, within the framework of the large-N model, the existence of three distinct time regimes. Symmetry being broken from the outset we have analysed the behaviour of transverse fluctuations, finding that in the early and intermediate stages these essentially evolve in the same way as in a symmetrical quench. This indicates that the growth of magnetization is a slow process, while growth of transverse fluctuations is much faster. Then, as long as magnetization is small the system also attempts to develop a Bragg peak in the transverse directions by condensation of fluctuations at $\boldsymbol{k} = 0$. This process eventually stops and regresses when $m(t)$ becomes of the order of the saturation value. According to this picture, the growth of order in the symmetrical quench is a particular case where condensation of the Nambu–Goldstone modes at $k = 0$ continues over an unbounded time interval. Different behaviour is then to be expected when the order parameter is conserved, since in that case condensation at $k = 0$ is prevented by the conservation law.

Finally, ordering along directions perpendicular to that of symmetry breaking is a phenomenon of more general occurrence than that described here, having also being found in a rather different physical situation, such as the response of a system subjected to a time-varying magnetic field [11].

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